B.Sc. Part II (Hons.) 4^{th} Paper

$\boxed{\text{LDE}}$

Q: $(D^2+1)y = \cos x \cdot \sin 3x$

Soln.

For CF, $D^2+1=0 \Rightarrow D = \pm i$

$\therefore CF = c_1 e^{ix} + c_2 e^{-ix} = c_1(\cos x + i\sin x) + c_2(\cos x - i\sin x)$

$\Rightarrow CF = (c_1+c_2)\cos x + i(c_1-c_2)\sin x$

$\Rightarrow CF = A\cos x + B\sin x$.

Now, $PI = \dfrac{1}{D^2+1} \cos x \sin 3x$

$\Rightarrow PI = \dfrac{1}{2(D^2+1)} 2\cos x \sin 3x$

$\Rightarrow PI = \dfrac{1}{2(D^2+1)} [\sin(3x+x) + \sin(3x-x)]$

$= \dfrac{1}{2(D^2+1)} (\sin 4x + \sin 2x)$

$= \dfrac{1}{2(D^2+1)} \sin 4x + \dfrac{1}{2(D^2+1)} \sin 2x$

$= \dfrac{1}{2\cdot(-4^2+1)} \sin 4x + \dfrac{1}{2(-2^2+1)} \sin 2x$

$\Rightarrow \quad PI = -\frac{1}{30}\sin 4x - \frac{1}{6}\sin 2x.$

$\therefore$ Complete soln is given by

$$y = CF + PI$$

$$\Rightarrow y = A\cos x + B\sin x - \frac{1}{30}\sin 4x - \frac{1}{6}\sin 2x.$$

Q. Solve $(D^2+4)y = \sin^2 x$

Soln. For CF, $D^2+4=0 \Rightarrow D = \pm 2i$

$$\therefore CF = c_1 e^{2ix} + c_2 e^{-2ix} = (c_1+c_2)\cos 2x + (c_1-c_2)i\sin 2x$$

$$\Rightarrow CF = A\cos 2x + B\sin 2x;\quad A = c_1+c_2,\quad B = (c_1-c_2)i$$

Now, we find PI.

$$PI = \frac{1}{D^2+4}\sin^2 x = \frac{1}{2(D^2+4)}(1-\cos 2x)$$

$$\Rightarrow PI = \frac{1}{2(D^2+4)}\cdot 1 - \frac{1}{2(D^2+4)}\cos 2x$$

$$\Rightarrow PI = \frac{1}{2(D^2+4)}e^{0x} - \frac{1}{2}\cdot\frac{x}{2}\int\cos 2x\,dx$$

$$= \frac{1}{2\cdot(0+4)}\cdot 1 - \frac{1}{2}\times\frac{x}{2}\,\frac{\sin 2x}{2}$$

$$\Rightarrow PI = \frac{1}{8} - \frac{x}{8}\sin 2x.$$

$\therefore$ Complete soln is $\quad y = CF + PI$

$$\Rightarrow y = A\cos 2x + B\sin 2x + \frac{1}{8} - \frac{x}{8}\sin 2x.$$